Mark Scheme (Results)

January 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01R

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Types of mark
- M marks: method marks
- A marks: accuracy marks - can only be awarded when relevant $M$ marks have been gained
- B marks: unconditional accuracy marks (independent of M marks)


## - Abbreviations

- cao - correct answer only
- cso - correct solution only
- ft - follow through
- isw - ignore subsequent working
- SC - special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- awrt - answer which rounds to
- eeoo - each error or omission
- No working

If no working is shown then correct answers may score full marks
If no working is shown then incorrect (even though nearly correct) answers score no marks.

- With working

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)
If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used
Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.
It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- Parts of questions

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

## Method mark for solving a 3 term quadratic equation:

## 1. Factorisation:

$$
\begin{aligned}
& \left(x^{2}+b x+c\right)=(x+p)(x+q) \text {, where }|p q|=|c| \text { leading to } x=\ldots \\
& \left(a x^{2}+b x+c\right)=(m x+p)(n x+q) \text { where }|p q|=|c| \text { and }|m n|=|a| \text { leading to } x=\ldots .
\end{aligned}
$$

2. Formula:

Attempt to use the correct formula (shown explicitly or implied by working) with values for $a, b$ and $c$, leading to $x=\ldots$.

## 3. Completing the square:

$$
x^{2}+b x+c=0:\left(x \pm \frac{b}{2}\right)^{2} \pm q \pm c=0, \quad q \neq 0 \quad \text { leading to } x=\ldots
$$

## Method marks for differentiation and integration:

## 1. Differentiation

Power of at least one term decreased by 1. $\left(x^{n} \rightarrow x^{n-1}\right)$
2. Integration:

Power of at least one term increased by 1. $\left(x^{n} \rightarrow x^{n+1}\right)$

## Use of a formula:

Generally, the method mark is gained by either
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values
or, where the formula is not quoted, the method mark can be gained by implication
from the substitution of correct values and then proceeding to a solution.

## Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

## Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

## Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

## MARK SCHEME

| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 1 | $\begin{aligned} & \frac{(a+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}=\frac{2 a+\sqrt{3}(a+2)+3}{1} \\ & 2 a+\sqrt{3}(a+2)+3=11+b \sqrt{3} \Rightarrow 11=2 a+3, \quad b=a+2 \end{aligned}$ <br> Solves the equations in $a$ and $b \Rightarrow a=4, \quad b=6$ <br> ALT $\begin{aligned} & \frac{a+\sqrt{3}}{2-\sqrt{3}}=11+b \sqrt{3} \Rightarrow a+\sqrt{3}=(2-\sqrt{3})(11+b \sqrt{3}) \\ & \Rightarrow a+\sqrt{3}=(22-3 b)+(2 b-11) \sqrt{3} \\ & \Rightarrow a=22-3 b \text { and } 1=2 b-11 \end{aligned}$ <br> Solves the equations in $a$ and $b \Rightarrow a=4, \quad b=6$ | M1 <br> M1M1 <br> A1 <br> [4] <br> \{M1 \} <br> \{M1 \} \{M1 \} <br> \{A1\} <br> [4] |
| Total 4 marks |  |  |
| (a) M1 | Multiply by $\frac{(2+\sqrt{3})}{(2+\sqrt{3})}$ |  |
| M1 | For either $11=2 a+3$ or $b=a+2$ For $11=2 a+3$ and $b=a+2$ $a=4, \quad b=6$ |  |
| M1 |  |  |
| A1 |  |  |
| ALT |  |  |
| M1 | Multiply by $2-\sqrt{3}$ <br> For either $a=22-3 b$ or $1=2 b-11$ <br> For $a=22-3 b$ and $1=2 b-11$ $a=4, \quad b=6$ |  |
| M1 |  |  |
| M1 |  |  |
| A1 |  |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 2 (a) | $\begin{aligned} & 7+4 x-x^{2}=11-(x-2)^{2} \\ & {[a=11, b=1, c=-2]} \end{aligned}$ <br> ALT $\begin{aligned} & 7+4 x-x^{2}=a-b\left(x^{2}+2 c x+c^{2}\right) \\ & a-b c^{2}=7 \quad b=1 \quad b c=4 \quad \text { So } a=11, b=1, c=-2 \\ & 7+4 x-x^{2}=11-(x-2)^{2} \end{aligned}$ | M1A1A1 <br> [3] $\begin{gathered} \{\mathrm{M} 1\} \\ \{\mathrm{A} 1\}\{\mathrm{A} 1\} \end{gathered}$ [3] |
| (b) | $\begin{array}{lll}\text { (i) } & 11 \\ \text { (ii) } & 2\end{array}$ | $\begin{gathered} \text { B1 ft } \\ \text { B1 ft } \\ {[2]} \end{gathered}$ |
| Total 5 marks |  |  |
| (a) |  |  |
| M1 | An attempt to factorise to make $x^{2}$ positive e.g. $-(x \pm p)^{2} \pm q$ |  |
| A1 | Complete the square to obtain an expression in the form $-(x \pm 2)^{2} \pm q$ NB Any expression in this form will score M1A1 |  |
| A1 | $11-(x-2)^{2}$ or $a=11, b=1, c=-2$ |  |
| ALT |  |  |
| M1 | Expands $a-b(x+c)$ |  |
| A1 | $a-b c^{2}=7 \quad b=1 \quad b c=4$ |  |
| A1 | $11-(x-2)^{2}$ or $a=11, b=1, c=-2$ |  |
| (b) (i) | Mark parts b(i) and b(ii) together |  |
| B1ft | 11 follow through their a |  |
| (b) (ii) | 2 follow through their c |  |
| B1ft |  |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 3(a) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=2 \mathrm{e}^{2 x}\left(x^{2}+1\right)+\mathrm{e}^{2 x}(2 x)$ | $\begin{gathered} \text { M1A1A1 } \\ {[3]} \end{gathered}$ |
| (b) | When $x=0$ $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} x}=2 \times 1 \times 1+1 \times 0=2 \quad y=\mathrm{e}^{2 \times 0}(0+1)=1 \\ & y-1=2(x-0) \Rightarrow y=2 x+1 \end{aligned}$ | B1B1 <br> B1 <br> [3] |
| Total 6 marks |  |  |
| (a) |  |  |
| M1 | Attempted use of the product rule. Sum of two terms (either way round) with $x^{n} \rightarrow x^{n-1}$ (Condone $e^{2 x}$ instead of $2 e^{2 x}$ ) Once the correct answer is seen ISW. This mark may be implied by the sum of two terms with one of the two terms correct. |  |
| A1 | Either term correct |  |
| A1 <br> (b) | Both terms correct |  |
| B1 | When $x=0 \quad \frac{\mathrm{~d} y}{\mathrm{~d} x}=2$ |  |
| B1 | When $x=0 \quad y=1$ |  |
| B1 | $y=2 x+1$ |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 4 (a) | $\begin{aligned} & \mathrm{f}(2)=2 \times 2^{3}+a \times 2^{2}+b \times 2+18=0 \\ & \mathrm{f}^{\prime}(x)=6 x^{2}+2 a x+b \Rightarrow \mathrm{f}^{\prime}(2)=6 \times 2^{2}+2 \times a \times 2+b=5 \\ & 4 a+2 b+34=0 \\ & 4 a+b+19=0 \\ & \Rightarrow b=-15, a=-1 \end{aligned}$ | M1 M1M1 A1 M1A1 $[6]$ |
| (b) | $\begin{aligned} & x - 2 \longdiv { 2 x ^ { 2 } + 3 x - 9 } \\ & 2 x^{2}+3 x-9=(x+3)(2 x-3) \\ & \Rightarrow(x-2)(x+3)(2 x-3) \end{aligned}$ | M1 <br> M1A1 <br> [3] |
| (c) | $x=2,-3, \frac{3}{2}$ | $\begin{gathered} \text { B2ft } \\ {[2]} \\ \hline \end{gathered}$ |
| Total 11 marks |  |  |
| (a) |  |  |
| M1 | $\mathrm{f}(2)=0$ leading to an equation in $a$ and $b$ |  |
| M1 | Attempt to differentiate |  |
| M1 | $\mathrm{f}^{\prime}(2)=5$ leading to an equation in $a$ and $b$ |  |
| A1 | $4 a+2 b+34=0$ and $4 a+b+19=0$ |  |
| M1 | Solving simultaneously |  |
| A1 | $b=-15, a=-1$ |  |
| (b) |  |  |
| M1 | Dividing by $x-2$ to obtain a 3TQ |  |
| M1 | Factorising the 3TQ |  |
| A1 | All 3 terms correct |  |
| (c) | $x=2,-3, \frac{3}{2}$ <br> (B1 for 2 correct) |  |
| B2 ft |  |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 5 (a) | $\log _{4} 32=\frac{\log _{2} 32}{\log _{2} 4}=\frac{5}{2} *$ or $\log _{4} 32=\log _{4} 4^{\frac{5}{2}}=\frac{5}{2} *$ or $\log _{4} 32=\log _{2^{2}} 2^{5}=\frac{5}{2} *$ <br> ALT $\log _{4} 32=a \quad \Rightarrow 4^{a}=32 \Rightarrow a=\frac{5}{2} *$ | M1A1cso <br> [2] $\{\mathrm{M} 1\}\{\mathrm{A} 1\}$ <br> cso <br> [2] |
| (b) | $\log _{2} x-\log _{4} 32+\frac{1}{4} \log _{x} 16=0$ <br> Let $\log _{2} x=y$ $\begin{aligned} & y-\frac{5}{2}+\frac{1}{4}\left(\frac{\log _{2} 16}{\log _{2} x}\right)=0 \quad \text { or } \quad y-\frac{5}{2}+\frac{1}{\log _{2} x}=0 \\ & \Rightarrow y-\frac{5}{2}+\frac{1}{y}=0 \\ & \Rightarrow 2 y^{2}-5 y+2=0 \\ & \Rightarrow(2 y-1)(y-2)=0 \\ & \Rightarrow y=\log _{2} x=\frac{1}{2} \text { or } 2 \\ & \Rightarrow x=2^{\frac{1}{2}}=\sqrt{2} \quad \text { and } \quad x=2^{2}=4 \end{aligned}$ | M1A1 <br> M1 <br> M1 <br> M1A1 <br> [7] |
| Total 9 marks |  |  |
| (a) M1 | For $\log _{4} 32=\frac{\log _{2} 32}{\log _{2} 4}$ or $\log _{4} 32=\log _{4} 4^{\frac{5}{2}}$ or $\log _{4} 32=\log _{2^{2}} 2^{5}$ |  |
| $\begin{gathered} \text { ALT } \\ \text { M1 } \end{gathered}$ | For $4^{a}=32$ |  |
| A1 cso <br> (b) | Obtains the given answer with no errors in the working |  |
| M1 | Use of $\log _{a} x=\frac{\log _{b} x}{\log _{b} a}$ or $\log _{a} b=\frac{1}{\log _{b} a}$ |  |
| M1 | Forming a 3TQ |  |
| A1 | $2 y^{2}-5 y+2=0$ |  |
| M1 | Solving the 3TQ |  |
| M1 | For $y=\log _{2} x=\frac{1}{2}$ or 2 |  |
| M1 | Either $x=2^{\frac{1}{2}}=\sqrt{2}$ or $x=2^{2}=4$ |  |
| A1 | Both $x=2^{\frac{1}{2}}=\sqrt{2} \quad$ and $\quad x=2^{2}=4$ |  |



| (a) |  |
| :--- | :--- |
| B2 | All 4 points correct |
| (b) | (B1 for 3 points correct $)$ |
| B1ft | Points plotted ft their table allow half a square tolerance |
| B1ft | Points joined together with a smooth curve ft their table |
| (c) |  |
| M1 | Setting $x-\frac{3}{x^{2}}=a x+b$ and simplifying to $x^{3}(a-1)+b x^{2}+3$ |
| M1 | Comparing coefficients |
| A1 | Identifying that the line required is $y=3 x-6$ |
| M1 | $y=3 x-6$ drawn intersecting the curve in two places |
| A1 | $x=0.8 / 0.9$ and $x=2.8 / 2.9$ |
| ALT |  |
| M1 | Subtracting 3 from both sides and dividing by $x^{2}$ |
| M1 | Adding $x$ to both sides |
| A1 | Identifying that the line required is $y=3 x-6$ |
| M1 | $y=3 x-6$ drawn intersecting the curve in two places |
| A1 | $x=0.8 / 0.9$ and $x=2.8 / 2.9$ |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 7 (a)(i) (ii) | $\begin{aligned} & a+4 d=4 x+6 \\ & a+7 d=7 x+3 \\ & \Rightarrow 3 d=3 x-3 \Rightarrow d=x-1^{*} \\ & a+7(x-1)=7 x+3 \Rightarrow a=10 \quad \text { or } \quad a+4(x-1)=4 x+6 \Rightarrow a=10 \end{aligned}$ | M1A1cso <br> M1A1 <br> [4] |
| (b) | $42=10+8(x-1) \Rightarrow x=5$ | $\begin{gathered} \text { M1A1 } \\ {[2]} \\ \hline \end{gathered}$ |
| (c) | $\begin{aligned} & d=x-1 \Rightarrow d=5-1=4 \\ & S_{n+1}=12 U_{n}+18 \Rightarrow \frac{n+1}{2}(2 \times 10+[(n+1)-1] 4)=12[10+(n-1) 4]+18 \\ & \Rightarrow n^{2}-18 n-40=0 \\ & \Rightarrow(n-20)(n+2)=0 \Rightarrow n=20 \end{aligned}$ | $\begin{gathered} \text { B1 } \\ \text { M1 } \\ \text { M1 } \\ \text { M1A1 } \\ {[5]} \\ \hline \end{gathered}$ |
| Total 11 marks |  |  |
| (a) (i) |  |  |
| M1 |  |  |
| A1 cso | Obtains the given answer with no errors in the working |  |
| (a) (ii) |  |  |
| M1 | Substitution of $d=x-1$ |  |
| A1 | $a=10$ |  |
| (b) M1 |  |  |
| A1 | $x=5$ |  |
| (c) |  |  |
| B1 | $d=4$ |  |
| M1 | Use of $\frac{n}{2}(2 a+(n-1) d)$ |  |
| M1 | Simplifying to $n^{2}-18 n-40=0$ |  |
| M1 | Solving the 3TQ <br> $n=20$ if shown must reject $n=-2$ |  |
| A1 |  |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 8 | $s=\int\left(3+5 t-2 t^{2}\right) \mathrm{d} t=3 t+\frac{5 t^{2}}{2}-\frac{2 t^{3}}{3}+c$ <br> when $t=0 \quad s=5$ $\begin{aligned} & 5=0+0-0+c \\ & s=5+3 t+\frac{5 t^{2}}{2}-\frac{2 t^{3}}{3} \end{aligned}$ <br> When $s=x$ $\begin{aligned} & \frac{\mathrm{d} x}{\mathrm{~d} t}=3+5 t-2 t^{2}=0 \Rightarrow(2 t+1)(t-3)=0 \Rightarrow t=3 \\ & \Rightarrow x=5+3 \times 3+\frac{5 \times 3^{2}}{2}-\frac{2 \times 3^{3}}{3}=\frac{37}{2} \text { oe } \\ & \frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}=5-4 t \text { when } t=3 \frac{\mathrm{~d}^{2} x}{\mathrm{~d} t^{2}}=-7 \Rightarrow \max \end{aligned}$ | M1 <br> B1 <br> A1 <br> M1A1 <br> A1 <br> M1A1 <br> [8] |
| Total 8 marks |  |  |
| M1 | Attempt to integrate |  |
| B1 | $c=5$ |  |
| A1 | $s=5+3 t+\frac{5 t^{2}}{2}-\frac{2 t^{3}}{3}$ |  |
| M1 | Solving $3+5 t-2 t^{2}=0$ |  |
| A1 | $t=3$ if shown must reject $t=-\frac{1}{2}$ |  |
| A1 | $x=\frac{37}{2} \text { oe }$ |  |
| M1 | Differentiates to obtain $\left(\frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}=\right) 5-4 t$ |  |
| A1 | Establish that the maximum has been obtained and give a conclusion |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 9 (a) | $a=-1, b=-2$ | $\begin{gathered} \text { B1,B1 } \\ {[2]} \\ \hline \end{gathered}$ |
| (b) | Gradient of $l_{1}=-2, \Rightarrow$ Gradient of $l_{2}=\frac{1}{2}$ $\begin{aligned} 180 & =(x+1)^{2}+(y-6)^{2} \\ \frac{1}{2} & =\frac{y-6}{x+1} \Rightarrow x=2 y-13 \end{aligned}$ <br> Solves simultaneous equations; $\begin{aligned} & 180=([2 y-13]+1)^{2}+(y-6)^{2} \Rightarrow 0=5 y^{2}-60 y \\ & \text { or } \quad 180=(x+1)^{2}+\left(\frac{1}{2} x+\frac{13}{2}-6\right)^{2} \Rightarrow 0=x^{2}+2 x-143=0 \\ & y=0, y=12 \Rightarrow x=-13, x=11 \quad \text { or } \quad x=-13, x=11 \Rightarrow y=0, y=12 \end{aligned}$ <br> Coordinates are $(-13,0)$ and $(11,12)$ | B1,B1 <br> M1 <br> M1 <br> M1M1 <br> A1A1 <br> [8] |
| (c) | Area of triangle $P Q R$ $\begin{aligned} & P Q=\sqrt{(6+2)^{2}+(-1-3)^{2}}=4 \sqrt{5} \\ & \text { Area } \left.=\frac{1}{2} \times 4 \sqrt{5} \times 6 \sqrt{5}=60 \text { (units }\right)^{2} \end{aligned}$ <br> ALT $\begin{aligned} & \text { Area }=\frac{1}{2}\left(\begin{array}{cccc} -13 & -1 & 3 & -13 \\ 0 & 6 & -2 & 0 \end{array}\right)=\frac{1}{2}(-78+2+0-0-18-26)=-60 \\ & \Rightarrow 60 \text { (units) }^{2} \end{aligned}$ | M1 <br> M1A1 <br> [3] <br> \{M1\} \{M1\} <br> \{A1 \} <br> [3] |
| (d) | Coordinates of $R$ required are $(-13,0)$ $\angle R P Q=90^{\circ}$ so $R Q$ is a diameter $\left(\frac{-13+3}{2}, \frac{0-2}{2}\right) \Rightarrow(-5,-1)$ | $\begin{gathered} \text { M1A1 } \\ {[2]} \end{gathered}$ |
| Total 15 marks |  |  |
| $\begin{aligned} & \hline \text { (a) } \\ & \text { B1 } \\ & \text { B1 } \\ & \hline \end{aligned}$ | $\begin{aligned} & a=-1 \\ & b=-2 \end{aligned}$ |  |


| (b) |  |
| :---: | :---: |
| B1 | Gradient of $l_{1}=-2$ |
| B1 | Gradient of $l_{2}=\frac{1}{2}$ |
| M1 | Use of $P R=6 \sqrt{5}$ to obtain an equation |
| M1 | Use of gradient of the perpendicular to obtain an equation |
| M1 | Solves simultaneously |
| M1 | Simplifies to $5 y^{2}-60 y=0$ or $x^{2}+2 x-143=0$ |
| A1 | All 4 values identified $0,12,-13,11$ |
| A1 | $(11,12)$ and $(-13,0)$ (must be paired correctly and if written as a coordinate then must be in the correct order) |
| (c) |  |
| M1 | $P Q=4 \sqrt{5}$ |
| M1 | Use of Area $=\frac{1}{2} \times P Q \times P R$ |
| A1 | 60 (units) ${ }^{2}$ |
| ALT |  |
| M1 | Use of Area $=\frac{1}{2}\left(\begin{array}{cccc}-13 & -1 & 3 & -13 \\ 0 & 6 & -2 & 0\end{array}\right) \mathrm{ft} R$ provided $e<0$ |
| M1 | $\frac{1}{2}(-78+2+0-0-18-26) \mathrm{ft} R$ provided $e<0$ |
| A1 (d) | 60 (units) ${ }^{2}$ |
| M1 | $\left(\frac{-13+3}{2}, \frac{0-2}{2}\right) \mathrm{ft} R$ provided $e<0$ |
| A1 | $(-5,-1)$ |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 10 (a) | $A B=A O+O B=-(2 \mathbf{a}-\mathbf{b})+3 \mathbf{a}+\mathbf{b}=\mathbf{a}+2 \mathbf{b}$ | $\begin{gathered} \text { M1A1 } \\ \hline \end{gathered}$ |
| (b) | $O C=O B+B C=3 \mathbf{a}+\mathbf{b}-\mathbf{a}+3 \mathbf{b}=2 \mathbf{a}+4 \mathbf{b}=2(\mathbf{a}+2 \mathbf{b})$ <br> Conclusion required; same direction and $\stackrel{\text { とnu }}{O} C$ is a multiple of $\stackrel{\text { «um }}{A M}$ therefore $\stackrel{\text { unu }}{O C}$ is parallel to $A B$. | M1 <br> A1 <br> [2] |
| (c) | $\begin{aligned} & A C=A B+B C=\mathbf{a}+2 \mathbf{b}+(-\mathbf{a}+3 \mathbf{b})=5 \mathbf{b} \\ & \operatorname{unu}_{A X}=\mu A C=\mu 5 \mathbf{b} \\ & \operatorname{unur}_{\operatorname{unu}} \operatorname{unu} \operatorname{unu} \\ & A X=A O+O X=-(2 \mathbf{a}-\mathbf{b})+\lambda(\mathbf{3 a}+\mathbf{b}) \\ & \Rightarrow \mu 5 \mathbf{b}=-(2 \mathbf{a}-\mathbf{b})+\lambda(\mathbf{3 a}+\mathbf{b}) \\ & \Rightarrow-2+3 \lambda=0 \Rightarrow \lambda=\frac{2}{3} \\ & \Rightarrow 5 \mu=1+\lambda \Rightarrow \mu=\frac{1}{3} \\ & \Rightarrow A X: X C=1: 2 \end{aligned}$ | B1 <br> B1 <br> M1 <br> M1 <br> M1 <br> A1 <br> A1 <br> [7] |
| Total 11 marks |  |  |
| (a) | Use of $\stackrel{\text { unu }}{A B}=\stackrel{\text { unu }}{A M}+\stackrel{\text { unu }}{O}$ |  |
| M1 |  |  |
| A1 | $\mathbf{a}+2 \mathbf{b}$ |  |
| M1 | Use of $\stackrel{\text { unu }}{O C}=\stackrel{\text { unum }}{O B}+B C$ |  |
| A1 | Correct conclusion i.e. $\stackrel{\text { unu }}{O} C=2 \underset{A}{\text { unu }} \quad \therefore \stackrel{\text { unu }}{O}$ is parallel to $\stackrel{\text { unu }}{A B}$ |  |
| (c) | una |  |
| B1 | $\underset{\text { cuum }}{A C}=5 \mathbf{b}$ may be eimplied by $2^{\text {nd }} \mathrm{B} 1$ |  |
| B1 | $A X=\mu 5 \mathbf{b}$ or $X C=\lambda 5 \mathbf{b}$ |  |
| M1 | A correct vector for $A X$ or $O X$ or $B X$ or $C X$ or $B C$ including an unknown multiple of a vector e.g. $A X=-(2 \mathbf{a}-\mathbf{b})+\lambda(3 \mathbf{a}+\mathbf{b})$ |  |
| M1 | Equate 2 forms of the same vector e.g. $\mu \mathbf{5 b}=-(2 \mathbf{a}-\mathbf{b})+\lambda(\mathbf{3 a}+\mathbf{b})$ |  |
| M1 | Comparing coefficients |  |
| A1 | $\lambda=\frac{2}{2} \text { or } \mu=\frac{1}{2}$ |  |
| A1 | $A X: X C=1: 2$ |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 11 (a) | at $P \quad b=\sqrt{a-2} \Rightarrow b^{2}=a-2 *$ | M1A1cso <br> [2] |
| (b) | $\begin{aligned} & \text { At } P \quad y=b \Rightarrow y^{2}=b^{2} \Rightarrow y^{2}=a-2 \\ & V=\pi \int_{a}^{16}(\sqrt{x-2})^{2} \mathrm{~d} x-\pi \int_{a}^{16}(a-2) \mathrm{d} x=\pi \int_{a}^{16}(x-2) \mathrm{d} x-\pi \int_{a}^{16}(a-2) \mathrm{d} x \\ & \Rightarrow \pi \int_{a}^{16}(x-a) \mathrm{d} x \quad \text { or } \quad \pi \int_{a}^{16}(\sqrt{x-2})^{2} \mathrm{~d} x-\pi(a-2)(16-a) \\ & 50 \pi=\pi\left[\frac{x^{2}}{2}-a x\right]_{a}^{16} \quad \text { or } \quad 50 \pi=\pi\left[\frac{x^{2}}{2}-2 x\right]_{a}^{16}-\pi(a-2)(16-a) \\ & 50 \pi=\pi\left[\left(\frac{256}{2}-16 a\right)-\left(\frac{a^{2}}{2}-a^{2}\right)\right] \\ & \text { or } 50 \pi=\pi\left[\left(96-\frac{a^{2}}{2}+2 a\right)-\left(18 a-a^{2}-32\right)\right] \\ & \Rightarrow a^{2}-32 a+156=0 \\ & \Rightarrow(a-6)(a-26)=0 \Rightarrow a<16 \text { so } a=6 \\ & b^{2}=a-2 \Rightarrow b^{2}=4 \Rightarrow b=2 \end{aligned}$ | B1 <br> M1 <br> depM1A1 <br> depM1 <br> M1 <br> M1A1 <br> A1 <br> [9] |
| Total 11 marks |  |  |
| (a) | $b=\sqrt{a-2}$ <br> Obtains the given answer with no errors in the working $y^{2}=a-2$ <br> Use of $V=\pi \int_{a}^{16}(\sqrt{x-2})^{2} \mathrm{~d} x-\pi \int_{a}^{16}(a-2) \mathrm{d} x$ or $V=\pi \int_{a}^{16}(x-a) \mathrm{d} x$ or $\pi \int_{a}^{16}(\sqrt{x-2})^{2} \mathrm{~d} x-\pi(a-2)(16-a)$ Ignore limits <br> Attempts to integrate. Ignore limits (Dependent on previous M1) Correct integration. Ignore limits <br> Correct substitution of limits (Dependent on previous M1) <br> Obtaining the 3TQ <br> Solving the 3TQ $\begin{aligned} & a=6 \\ & b=2 \\ & \hline \end{aligned}$ |  |
|  |  |  |
| A1 cso <br> (b) |  |  |
| B1 |  |  |
| M1 |  |  |
| depM1 |  |  |
| A1 |  |  |
| depM1 |  |  |
| M1 |  |  |
| M1 |  |  |
| A1 |  |  |
| A1 |  |  |

